NATIONAL CONFERENCE ON OPERATOR THEORY AND FUNCTION SPACES
Shiv Nadar IoE Delhi NCR, March 27-29, 2024
Abstracts
Contributory talks

Parallel session 1- 27th March 2024

1. Chaman Sahu

   TITLE: Helson matrices induced by positive Radon measures

   Abstract : A Helson matrix is an infinite matrix whose entry $a_{m,n}$ depends on the product mn. A prime example is the multiplicative Hilbert matrix $\mathcal{M}$ given by

   $$\mathcal{M} := \Big(\frac{1}{\sqrt{mn}\log(mn)}\Big)_{m,n=2}^{\infty}.$$

   In this talk, we introduce a class of Helson matrices induced by a Radon measure μ, not necessarily finite, on $(0,\infty)$ and discuss their boundedness, Schatten-class membership and spectral properties.

   This talk is based on an ongoing work with Prof. K. B. Sinha and Prof. Sameer Chavan.

2. Jyoti Rani

   TITLE: Spectral Analysis of infinite matrices generated by non-convergent sequences

   Abstract: In this article, we study the spectral properties of a specific class of infinite band matrices. A band matrix is a type of sparse matrix where most of the elements are zero, except for those along a diagonal band and possibly a few additional bands parallel to it. The width of the band (the number of non-zero diagonals) determines the bandwidth of the matrix. Researchers worldwide have extensively explored the spectral analysis of infinite matrices, with a specific focus on band matrices defined over sequence spaces.

   In the past few decades, there has been a significant evolution in the study of the localization of spectrum and various subdivisions of the spectrum in band matrices over sequence spaces. This has become a substantial area of investigation

within the field of spectral theory. The fine spectrum of the difference operator over the sequence spaces c_0(set of all null sequences) and c(set of all convergent sequences) has been studied by Altay and Basar. More recently, the fine spectra of the difference operator over the sequence spaces l_p(set of all p-absolutely summable sequences) have been studied by Akhmedov and Basar where p is a real number greater or equal to 1.

In our review of existing literature, we identified a gap in the research regarding the spectral properties of infinite matrices where the bands are formed by non-convergent sequences. Thus, this study aims to address and bridge this gap by investigating the spectral properties of such matrices. In this article, we have studied the spectral properties of particular type of band matrices where band contains non-convergent sequences. We have obtained fine spectrum of operator. Also, we use a significant theory of difference equations and the Fredholm operator. Moreover, we have used the compact perturbation technique to find the spectrum. Moreover, we have obtained some other subdivisions of the spectrum such as defect spectrum, discrete spectrum, compression spectrum, etc.

Also, with the help of singular values of a matrix we have obtained some important conditions for the essential spectrum. This study holds significance in that by specifying particular values for the bands, we can deduce the spectral properties of certain matrices, which are already studied in the literature, as special cases. Therefore, our study provides a more comprehensive and generalized version of these spectral properties.

3. Pintu Bhunia

TITLE: Numerical radius bounds via polar decomposition of operators

Abstract: Using the polar decomposition of a bounded linear operator defined on a complex Hilbert space, we obtain several numerical radius bounds, which improve and generalize the earlier related bounds. It is shown that

$$w(A) \leq \frac{1}{2}\|A\|^{1/2} \left\| |A|^t + |A^*|^{1-t} \right\|, \quad \textit{for all } t \in [0,1],$$

where $w(A)$ and $\|A\|$ are the numerical radius and the operator norm of an operator A, respectively.

The talk is based on the following paper:

[i] Pintu Bhunia, *Improved bounds for the numerical radius via polar decomposition of operators*, *Linear Algebra Appl.* 683 (2024), 31–45.

Parallel session 2- 27th March 2024

1. Samir Panja

   TITLE: Isometric dilation and commutant lifting of operator tuples with defect zero

   Abstract: In this talk, we will deal with isometric dilation and commutant lifting for a class of n-tuples ($n \geq 3$) of commuting contractions. We will show that operator tuples in the class dilate to tuples of commuting isometries of BCL type. As a consequence of such an explicit dilation, we will provide that their von Neumann inequality holds on a one dimensional variety of the closed unit polydisc. On the basis of such a dilation, we will prove a commutant lifting theorem of Sarason's type by establishing that every commutant can be lifted to the dilation space in a commuting and norm preserving manner. This further leads to find yet another class of n-tuples ($n \geq 3$) of commuting contractions each of which possesses isometric dilation. The content of the talk is based on a joint work with B. K. Das.

2. Krishna Kumar Gupta

   TITLE: Pólya algorithm in $\mathbb{C}^n$

   Abstract: Let $\mathcal{M}$ be a proper subspace of $\mathbb{R}^n$. Let $\mathbf{b} \in \mathbb{R}^n \setminus \mathcal{M}$. For $1 < p < \infty$, we call $\mathbf{x}_p$ as the ℓ_p-approximation of $\mathbf{b}$, if

   $$\|\mathbf{x}_p - \mathbf{b}\|_p = \min_{\mathbf{x} \in \mathcal{M}} \|\mathbf{x} - \mathbf{b}\|_p,$$

   where $\|\cdot\|_p$ denotes the ℓ_p norm. In [J. R. Rice, Tchebycheff approximation in a compact metric space, Bull. Amer. Math. Soc. 68 (1962), 405–410], the *strict Chebyshev approximation* of $\mathbf{b}$ was defined. Later, in [J. Descloux, Approximation in L^p and Chebyshev approximation, J. Soc. Indust. Appl. Math. 11 (1963), 1017–1026], the convergence of ℓ_p-solutions to the strict Chebyshev approximation as $p \to \infty$ was shown. This is called the *Pólya algorithm*. In [Marano, M. Strict approximation on closed convex sets. Approx. Theory its Appl. 6, 99–109 (1990)], the result has been extended over the closed convex sets of $\mathbb{R}^n$, which satisfy certain approximative properties. In [M. Marano, J. N. Ureña, The linear discrete Pólya algorithm, Appl. Math. Lett. 8 (1995), 25–28], the rate of convergence in $\mathbb{R}^n$ was given as $\frac{1}{p}$. In this talk, I shall indicate how the technique of Marano and Ureña can be extended to a subspace of $\mathbb{C}^n$.

3. Bharti Garg

   TITLE: J-contractive operator valued functions and vector valued de Branges spaces

   Abstract: This talk aims to give an overview of the reproducing kernel Hilbert spaces (RKHS) constructed from J-contractive operator valued analytic functions

on the upper half-plane. First we discuss the Potapov-Ginzburg transform of certain class of bounded linear operators, investigate certain properties and then present sufficient conditions for a J-contractive operator to be J-bicontractive. Finally, a construction of RKHS of vector valued analytic functions is proposed with the help of J-contractive operator valued analytic functions.
This is part of an ongoing work with my Ph.D. supervisor Dr. Santanu Sarkar.

References

[1] H. Dym, *Two classes of vector valued de Branges spaces,* J. Funct. Anal. 31 (2023) 109758.

[2] S. Mahapatra, S. Sarkar, *Vector valued de Branges spaces of entire functions based on pairs of Fredholm operator valued functions and functional model*, J. Math. Anal. Appl. 533 (2) (2024) 128010.

[3] B. Garg, S. Sarkar, *J-contractive operator valued functions, vector valued de Branges spaces and functional model*, Preprint.

Parallel session 1- 28th March 2024

1. Satish Pandey

   TITLE: Distance between reproducing kernel Hilbert spaces and geometry of finite sets in the unit ball

   Abstract : In this talk, we present, in a quantitative manner, the relationships between the structure of a reproducing kernel Hilbert space, the structure of its multiplier algebra, and the geometry of the underlying set. We introduce a variant of the Banach-Mazur distance suited for measuring the distance between reproducing kernel Hilbert spaces, that quantifies how far two spaces are from being isometrically isomorphic as reproducing kernel Hilbert spaces. We introduce an analogous distance for multiplier algebras, that quantifies how far two algebras are from being completely isometrically isomorphic. Using these notions of distances at our disposal, we show that, in the setting of finite dimensional quotients of the Drury-Arveson space, two spaces are "close" to one another if and only if their multiplier algebras are "close", and that this happens if and only if one of the underlying point sets is close to an image of the other under a biholomorphic automorphism of the unit ball. These equivalences are obtained as corollaries of quantitative estimates that we prove. This is joint work with Danny Ofek and Orr Moshe Shalit.

2. Satyabrata Majee

   TITLE: On Decomposition for Pairs of Twisted Contractions

Abstract: In this talk, we present Wold-type decomposition for various pairs of twisted contractions on Hilbert spaces. As a consequence, we obtain a new and simple proof of Słoćinski's theorem for pairs of doubly commuting isometries and generalized that result for pairs of doubly twisted isometries. We also achieve an explicit decomposition for pairs of twisted contractions such that the c.n.u. parts of the contractions are in C_{00}. It is also shown that for a twisted pair (T, V^*) of operators with T as a contraction and V as an isometry, there exists a unique (upto unitary equivalence) pair of doubly twisted isometries on the minimal isometric dilation space of T. As an application, we provide a new proof for pairs of twisted operators consisting of an isometry and a co-isometry are doubly twisted. This is a joint work with Amit Maji.

3. Arshad Khan

   TITLE: Nearly Invariant Brangesian Subspaces

   Abstract: In this talk, I will discuss my work on nearly invariant subspaces. Nearly invariant subspaces for backward shift operator on Hardy space $H^2(\mathbb{D})$ were first discussed by Hitt in his work on invariant subspaces of H^2 on an annulus. Since then, this work has been extended in many directions. Here, I will present our extensions of Hitt's theorem as well as its generalizations to the de Branges setting.

   This is joint work with Prof. Sneh Lata and Prof. Dinesh Singh.

4. Shubham Jain

   TITLE: Toeplitz operators on the n-dimensional Hartogs triangle

   Abstract: In this talk, we formally introduce and study Toeplitz operators on the Hardy space of the n-dimensional Hartogs triangle

   $$\triangle_0^n := \{(z_1, z_2, \ldots, z_n) \in \mathbb{C}^n : |z_1| < |z_2| < \ldots < |z_n| < 1\}.$$

   We find a precise relation between these operators and the Toeplitz operators on the Hardy space of the unit polydisc $\mathbb{D}^n$. As an application, we deduce several basic properties of these operators from their polydisc counterparts. This talk is based on a joint work with P. Pramanick.

5. Jaikishan

   TITLE: Gleason-Kahane-$\dot{Z}$elazko (GKZ) Theorem In Function Spaces

   Abstract: Gleason-Kahane-$\dot{Z}$elazko (GKZ) characterizes all the multiplicative linear functionals on a complex unital Banach algebras. Recently, the GKZ theorem has been extended by Javad Mashreghi and Thomas Ransford to function spaces that are not algebras.

In this talk, we present a general version of the GKZ theorem. First we characterize a class of linear functionals as point evaluations on the vector space of all complex polynomials $\mathcal{P}$. We then apply this characterization to present a version of the GKZ theorem for a vast class of topological spaces of complex-valued functions including the Hardy, Bergman, Dirichlet, and many well-known function spaces. Also, we use the GKZ theorem for polynomials to obtain a version of the GKZ theorem for strictly cyclic weighted Hardy spaces.

Parallel session 2- 28th March 2024

1. Bharat Talwar

   TITLE: UCP maps and multiplicativity

   Abstract : Let $\tau : \boldsymbol{M}_d \to \boldsymbol{M}_d$ be a unital completely positive map with spectrum inside the unit sphere of the complex plane. We will prove that τ respects multiplication.

2. Mohd. Shahbaz

   TITLE: Finite Strict Singularity, Co-singularity and Entropy Numbers of Laplace Transform

   Abstract: We consider the Laplace transform $\mathcal{L} : L^1(0,\infty) \to L^\infty(0,\infty)$ defined by $\mathcal{L}f(x) \coloneqq \int_0^\infty f(t)e^{-xt}dt, x \in (0,\infty)$ and compute its approximation quantities such as approximation, Kolmogorov and Gelfand numbers. Also, we explore the finite strict singularity and finite strict co-singulaity of $\mathcal{L}$ by estimating its Bernstein, Mityagin and isomorphism numbers. Furthermore, entropy numbers are computed and it is proved that $\mathcal{L}$ is maximally non-compact.

3. Shamim Sohel

   TITLE: Exploring some subspaces from the perspective of best coapproximation

   Abstract: We plan to study the best coapproximation problem in Banach spaces, by using Birkhoff-James orthogonality techniques. We discuss two recently introduced types of subspaces, named the anti-coproximinal subspaces and the strongly anti-coproximinal subspaces. A necessary condition for the strongly anti-coproximinal subspaces in a reflexive Banach space whose dual space satisfies the Kadets-Klee Property will be discussed. On the other hand, we provide a sufficient condition for the strongly anti-coproximinal subspaces in a general Banach space. We also characterize the anti-coproximinal subspaces of a smooth Banach space. Further, we study these special subspaces in a finite-dimensional polyhedral Banach space and find some interesting geometric structures associated with them.

4. Jekwin Dabhi

   TITLE: On the UC^*NP the of Lau product

   Abstract: A $*$- algebra $\mathcal{A}$ is a BG^*- *algebra* if for every $*$-representation π of $\mathcal{A}$ as linear operators on a pre-Hilbert space K, it is true that $\pi(a) \in B(K)$ for all $a \in A$, where $B(K)$ is the algebra of all bounded linear operators on K. A BG*-algebra $\mathcal{A}$ is *reduced* if the set of $*$- representations on $\mathcal{A}$ separates the points of $\mathcal{A}$, i.e., if $a \in \mathcal{A}$ and $\pi(a) = 0$ for every $*$- representation π on $\mathcal{A}$, then $a = 0$. C^*- *enveloping algebra* of $\mathcal{A}$ is the completion of $\mathcal{A}$ with respect to largest C^*- norm on $\mathcal{A}$. An algebra $\mathcal{A}$ is said to have UC*NP if it admits unique C* norm. This property has a significant role in harmonic analysis. It is shown that the Lau product $\mathcal{A} \times_\theta \mathcal{B}$ has unique C^*- norm property if and only if both $\mathcal{A}$ and $\mathcal{B}$ have unique C^*- norm property. Also characterization of the C^*- enveloping algebra of $\mathcal{A} \times_\theta \mathcal{B}$ will be presented.

5. Souvik Ghosh

   TITLE: The interplay between isosceles orthogonality and some geometric constants

   Abstract: The James constant in a Banach space $\mathbb{X}$ is defined as:

$$J(\mathbb{X}) = \sup\{\min\{\|x+y\|, \|x-y\|\} : \|x\| = \|y\| = 1\}.$$

   During this talk we describe how the notion of isosceles orthogonality plays a significant role regarding the attainment of James constant. We show that if $J(\mathbb{X}) = \min\{\|x+y\|, \|x-y\|\}$, then $x \perp_I y$. We prove that in a two-dimensional polyhedral Banach space there exists an extreme point z such that $J(\mathbb{X}) = \|z+y\| = \|z-y\|$, for some y with $\|y\| = 1$. Moreover, we study the approximate isosceles orthogonality and observe its connection with the modulus of convexity.