First Year Plan for B.Sc. (Research) Mathematics

Semester wise plan

Semester	Credits	Courses
Year 1 - Monsoon Semester	20	Foundations Calculus I Linear Algebra I Number Theory CCC1 EVS
Year 1 - Spring Semester	20	Calculus II Algebra I Statistical Thinking and Probability Introduction to Programming UWE1
Year 1 - Summer Semester	2	Mathematics in the Modern World I

First-year course catalogue names in the prospectus

- MAT 1000 - Foundations
- MAT 1001 - Calculus I
- MAT 1006 - Linear Algebra I
- MAT 1005 - Number Theory
- MAT 1002 - Calculus II
- MAT 2005 - Algebra I
- MAT 1007 - Statistical Thinking and Probability
- CSD XXXX - Introduction to Programming

MAT 1000 - Foundations

Syllabus / Curriculum Content

- Sentential logic: deductive reasoning, negation, conjunction and disjunction, equivalence, truth tables, logical connectives.
- Sets: operations on sets, Venn diagrams, Cartesian product, quantifiers.
- Proof strategies: direct proofs, proofs involving negations, conditionals, conjunctions and disjunctions, existence and uniqueness proofs, equivalence proofs.
- Relations: ordered pairs, equivalence relations, partial order, equivalence classes, partitioning of a set.
- Functions: functions as many-one relations, graphs, one-one functions, onto functions, inverse functions, images and inverse images of sets.

Books / References

- Daniel J. Velleman, How to Prove It, Cambridge University Press.
- Ethan D. Bloch, Proofs and Fundamentals, Springer.
- Amber Habib, Foundations.

MAT 1001 - Calculus I

Syllabus / Curriculum Content

- Real number system: axioms for N and R; mathematical induction.
- Integration: area as a set function, integration of step functions, upper and lower integrals, integrability of bounded monotone functions, basic properties of integration, polynomials, trigonometric functions.
- Continuous functions: functions, limits, continuity, Intermediate Value Theorem, Extreme Value Theorem, integrability of continuous functions, Mean Value Theorem for integrals.
- Differentiation: tangent line, rates of change, derivative as function, algebra of derivatives, implicit differentiation, related rates, linear approximation, inverse functions, standard functions, extrema, derivative tests, Rolle theorem, Mean Value Theorem, concavity, curve sketching.
- Fundamental Theorem of Calculus: antiderivatives, indefinite integrals, logarithm and exponential functions, techniques of integration.
- Polynomial approximations: Taylor polynomials, remainder formula, indeterminate forms and L Hopital rule, limits involving infinity, improper integrals.
- Ordinary differential equations: first-order separable, logistic growth, first-order linear equations, if time permits.

Books / References

- Tom M. Apostol, Calculus, Volume I, Wiley.
- Richard Courant and Fritz John, Introduction to Calculus and Analysis I, Springer.
- Jerrold Marsden and Alan Weinstein, Calculus I and II, Springer.
- G. F. Simmons, Calculus with Analytic Geometry, McGraw-Hill.
- James Stewart, Essential Calculus - Early Transcendentals, Cengage India Edition.
- MIT OpenCourseWare: Single Variable Calculus and Calculus with Theory.

MAT 1006 - Linear Algebra I

Syllabus / Curriculum Content

- Matrices and linear systems.
- Vector spaces and linear transformations.
- Inner product spaces.
- Eigenvalues, eigenvectors and diagonalization.
- Least-square approximation.

Books / References

- Ron Larson, Elementary Linear Algebra, Wadsworth Publishing Company.
- David C. Lay, Linear Algebra and its Applications, 3rd edition, Pearson.
- Sheldon Axler, Linear Algebra Done Right.
- Carl D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM.
- Gilbert Strang, Linear Algebra and its Applications, 4th edition, Cengage.

MAT 1005 - Number Theory

Syllabus / Curriculum Content

- First nine chapters of Elementary Number Theory by David M. Burton; remaining chapters may be used for presentations depending on student response.
- Mathematical induction and binomial theorem.
- Divisibility, gcd and linear Diophantine equations.
- Primes and their distribution.
- Theory of congruences.
- Fermat theorem and pseudoprimes.
- Number-theoretic functions, including Euler phi function.
- Euler generalization of Fermat theorem.
- Primitive roots and indices.
- Quadratic reciprocity law.
- Lab work: applying number-theory concepts in Python during lectures and tutorials.

Books / References

- David M. Burton, Elementary Number Theory.
- Gareth A. Jones, Elementary Number Theory.
- Kenneth H. Rosen, Elementary Number Theory and Its Applications.
- Ireland and Rosen, A Classical Introduction to Modern Number Theory.
- Udacity Programming Foundations with Python; Codecademy Learn Python; W3Schools Python tutorial.
- Mark Lutz, Learning Python, 5th edition; Luciano Ramalho, Fluent Python; Learn Python the Hard Way.

MAT 1002 - Calculus II

Syllabus / Curriculum Content

- Sequences and series: limits of sequences, series and sums, divergence test, comparison test, integral test.
- Parametric curves: Euclidean space and vectors, vector-valued functions, plane and space curves, tangent vectors and lines, polar/cylindrical/spherical coordinates, derivatives and integrals, arc length.
- Differential calculus in several variables: functions of several variables, level curves and surfaces, quadric surfaces, limits and continuity, partial derivatives, tangent planes, chain rule, directional derivatives, gradient.
- Applications to optimization: extreme values and saddle points, first and second derivative tests, Lagrange multipliers.
- Multiple integrals: double integrals, triple integrals and change of variables.

Books / References

- James Stewart, Essential Calculus - Early Transcendentals, Cengage India Edition.
- Tom Apostol, Calculus, Volumes I and II, 2nd edition, Wiley.
- J. E. Marsden, A. J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer India.
- Jerrold Marsden and Alan Weinstein, Calculus II and III, Springer.
- G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Addison-Wesley.
- Amber Habib, Calculus, Cambridge University Press.
- MIT OCW lecture videos: Single Variable Calculus by David Jerison and Multivariable Calculus by Denis Auroux.

MAT 2005 - Algebra I

Syllabus / Curriculum Content

- Groups: definition, examples and elementary properties; subgroups, subgroup tests, generated subgroups, cyclic groups, subgroups of cyclic groups, cosets and Lagrange theorem.
- Normal subgroups and quotient groups; homomorphisms, isomorphisms and automorphisms; conjugates, centre, centralizer, normalizer; Cayley theorem; direct products; finite abelian groups.
- Permutation groups: definition, examples and properties; symmetric group Sn; alternating group An.
- Group actions: orbit-stabilizer theorem, Cauchy theorem, Cayley theorem, Sylow theorems and applications.
- Use of GAP for basic group operations.

Books / References

- Joseph A. Gallian, Contemporary Abstract Algebra, 4th edition, Narosa, 1999.
- Michael Artin, Algebra, 2nd edition, Prentice Hall India, 2011.
- I. N. Herstein, Topics in Algebra, 2nd edition, Wiley India, 2006.
- John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson, 2003.
- Serge Lang, Undergraduate Algebra, 2nd edition, Springer India, 2009.
- David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd edition, John Wiley and Sons, 2011.
- G. Santhanam, Algebra, Narosa Publishing House.
- GAP documentation and online tutorials: https://docs.gap-system.org/doc/ref/chap0_mj.html and www.gap-system.org.

MAT 1007 - Statistical Thinking and Probability

Syllabus / Curriculum Content

- Module 1: data types and measurement scales; quantitative and qualitative data; nominal, ordinal, interval and ratio scales; frequency tables, bar graphs, pie charts, histograms, ogives and box plots.
- Module 2: descriptive statistics; mean, median, mode; range, quartile deviation, mean deviation, standard deviation; coefficient of variation; skewness and kurtosis.
- Module 3: relationships in data; scatter plots; simple, partial and rank correlation; least-squares principle and simple linear regression.
- Module 4: combinatorial techniques; counting principle, factorial notation and identities, permutations, combinations, circular permutations, multinomial coefficients.
- Module 5: probability; random experiments, sample space and events; classical, empirical and axiomatic probability; algebra of events; conditional probability and independence; addition and multiplication laws; total probability; Bayes theorem.
- Experiments/case studies: Excel-based data summarization and visualization; real-world datasets; correlation for economic/health indicators; virtual card and dice probability experiments.

Books / References

- A. Agresti and C. Franklin, Statistics: The Art and Science of Learning from Data, 4th edition, Pearson, 2017.
- D. C. Montgomery and G. C. Runger, Applied Statistics and Probability for Engineers, John Wiley and Sons, 2010.
- D. S. Moore, G. P. McCabe and B. A. Craig, Introduction to the Practice of Statistics, Vol. 4, W. H. Freeman, 2009.

CSD XXXX - Introduction to Computing and Programming

Syllabus / Curriculum Content

- Basic C programming constructs: conditional statements, functions, loops, arrays, structures and pointers.
- Linear data structures: linked list, queue and stack.
- Trees and graphs: basic operations.
- Searching and hashing: linear search, binary search, tree search and hash tables.
- Sorting: insertion sort, bubble sort, merge sort and heap sort.
- Introduction to MATLAB programming.

Books / References

- B. Kolman, R. Busby and S. Ross, Discrete Mathematical Structures, PHI, 2012.
- Jeri R. Hanly and Elliot B. Koffman, Problem Solving and Program Design in C, Pearson, 2009.
- A. Aho, J. Hopcroft and D. Ullman, Data Structures and Algorithms, Addison-Wesley, 1983.
- A. Aho and D. Ullman, Foundations of Computer Science, Computer Science Press, 1992.
- T. Cormen and C. Leiserson, Introduction to Algorithms, MIT Press, 2009.
- N. Kalicharan, Data Structures in C, CreateSpace Independent Publishing, 2008.
- A. Tenenbaum, Data Structures Using C, PHI, 2003.